TESSELLATE PRESENTS

STEMS

Scholastic Test of Excellence in Mathematical Sciences

Physics Category C

Exam Date	**:**	**16th December, 2023**
Exam Timing	**:**	**9:00 AM IST - 12:00 PM IST**

Rules and Regulations

Marking Scheme

1. The question paper contains **eleven** questions, **eight** multiple choice type objective questions (**Part A**), and **three** subjective questions (**Part B**).
2. Each objective question has a single correct option as the answer.
3. For each objective question, **4 marks** is given for the correct answer, **0 marks** is given for no attempt, and **-1 marks** is given for an incorrect answer. Each subjective question is worth **20 marks**.
4. A candidate's submission for the **Part B** of the exam will be checked only if they are in the top 15 candidates for **Part A**.
5. Time duration is **3 hours: 9:00 AM IST - 12:00 PM IST**.
   Submit your answers on the Google form given below by **12:20 PM IST**.

Miscellaneous

1. Use the google form: **https://forms.gle/s1SwGpPi7MgmetGA9** , to submit your answers.
2. For **Part B**, you can either LaTeX or handwrite your solutions neatly.
   Submit a PDF file (either scanned or LaTeXed) **ONLY**. No other form of file submission will be accepted. Name your file **"physics_rollnumber"** (here rollnumber is the last 5 digits of the Transaction ID generated at the time of registration).
3. Make sure to keep the file size below the 10 MB limit. You can use online file compression services in case your file size exceeds 10 MB.
4. Use a good application to scan handwritten text into PDF. Kindly make sure that the answers are legible and that your furniture or flooring is not a part of the submission.
5. Solutions should be brief and should contain all the necessary details. Ambiguous or illegible answers will not gain credits. If you strike something out, strike it out properly so that it is clear to the evaluator what you want to be read. Please avoid overwriting your answers.
6. Do **NOT** post/share the questions appearing in the contest on any forums or discussion groups while the contest is live. It will result in immediate disqualification of involved candidates when caught.
7. Answers should be your own and should reflect your independent thinking process. Any form of plagiarism or failure to comply with the aforementioned regulations may lead to disqualification.
8. Responses to the google form will not be accepted after the time is up.

Contact details

- For subject related queries, clearly mention your **category (C)** in the mail or WhatsApp text.
- For **subject related** queries, contact:
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Note: Use the personal emails only if the official email is unreachable. Use WhatsApp only if absolutely necessary, otherwise email is preferred.

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Questions

Part A

1. **[4 marks]** The range of motion of a pendulum with one rigid string with length L in 2d is a circle with area πL^2. Consider the range of motion of a pendulum attached to two non-rigid strings, both length L and distance d apart. What is the area of this range?

   Options: (a) πL^2 (b) $\pi\left(L^2 - \frac{d^2}{4}\right)$ (c) $\pi L\sqrt{L^2 - d^2}$ (Correct) (d) $\pi(L^2 - d^2)$

2. **[4 marks]** The Lagrangian for a system is $\mathcal{L} = \omega\frac{\dot{x}^2}{x^2} + \pi\frac{\dot{y}^2}{y^2}$. Find a conserved quantity other than the Lagrangian itself.

   Options: (a) $\omega\dot{x}^2 + \pi\dot{y}^2$ (b) $\omega\frac{\dot{x}}{x} + \pi\frac{\dot{y}}{y}$ (Correct) (c) $\omega\frac{\dot{x}}{x^2} + \pi\frac{\dot{y}}{y^2}$ (d) $2\omega\frac{\dot{x}^2}{x} + 2\pi\frac{\dot{y}^2}{y}$

3. **[4 marks]** 5 particles are placed on the corners of a cube one by one such that every new particle is placed an edge away from another particle. Find the number of possible states of the system.

   Options: (a) 1440 (Correct) (b) 384 (c) 24 (d) 96

4. **[4 marks]** In a 3-dimensional harmonic oscillator, the energy eigenvalues for each dimension are:

   $$\epsilon_i = \hbar\omega(n_i + 1/2)$$

   with $n_i = 0, 1, \ldots; i = x, y, z$ Hence the total energy becomes:

   $$E = \hbar\omega(n_x + n_y + n_z + 3/2)$$

   Let the total energy of the particle be $\frac{21}{2}\hbar\omega$. Find the number of possible ways this energy can be distributed among the three dimensions.

   Options: (a) 15 (b) 21 (c) 56 (d) 28 (Correct)

5. **[4 marks]** The Philosopher's Stone is an object that can turn metals into gold on touch. While no one has discovered something like that, we can synthesize Au-197 from metals like Bi-209 using fusion; in this case bombardment of C-12 atoms with projectile energy of 0.40 GeV/nucleon. Assume it takes one C-12 atom to break Bi-209 into Au-197. Calculate the energy equivalent left over for one nucleon reaction. Mass of one nucleon is 939 MeV.

   Options: (a) 28.2 GeV (b) 28.6 GeV (c) 22.9 GeV (Correct) (d) 22.5 GeV

6. **[4 marks]** Two boxes A and B are at the same temperature T and both contain the same number of Helium atoms. Which of the following is false?

Options:
(a) If the two systems are allowed to exchange particles, the direction in which most particles will flow depends only on the pressure.
(b) If the two systems are allowed to exchange heat, the net transfer will be 0.
(c) If A is allowed to expand into double the volume, and then the systems are allowed to exchange heat, it will move from B to A.
(d) If B is heated to 2T and the two systems are allowed to exchange particles, no particles move from A to B. (Correct)

In the next two questions, Choose the option closest to the final answer. Note the exact answer might not be an option.

7. **[4 mark]** A thin iron ring of radius 1 cm at a temperature of 350 K is wrapped around a rod of a stronger material (doesn't deform) and cooled down to room temperature (300 K). Find the amount of force exerted by the ring on the stronger material. (Coefficient of expansion of Fe = $12 \times 10^{-6}/K$, Modulus of elasticity = 200 MPa).

   Options: (a) 1×10^5 N (Correct) (b) 4×10^4 N (c) 1×10^4 N (d) 1×10^6 N

8. **[4 marks]** An electron at rest is bombarded with another moving with a momentum p creating an electron-positron pair. If as much of the kinetic energy is converted to the mass of the new particles, find the minimum required value for p in units of $m_e c$.

   Options: (a) 2 (b) $2\sqrt{3}$ (Correct) (c) $\sqrt{7}$ (d) 4

Part B

1. **[20 marks]** Model a cylinder (radius r, height h, mass M) bobbing in water under gravity as the same mass oscillating from a spring without gravity. Find the equivalent spring constant and natural length.

   Solution

   $$\begin{aligned} m\ddot{h} &= mg - \rho_w gAh \\ &= -\rho_w gA(h - m/\rho_w A) \end{aligned}$$

   where ρ_w is the density of water, A is the surface area of the bottom of the cylinder.
   So, the equivalent spring constant is $\rho_w gA$ and the natural length/equillibrium position is $m/\rho_w A$.

2. **[20 marks]** Study what happens to the momentum of a skateboarder when he moves on ramps of different shapes. (For simplicity, compare a straight ramp with a circular one).

   Solution

   The answers were graded according to whatever part was analysed.
   Some examples of things that could've been observed:

   - Getting onto the straight ramp cancels velocity proportional to the sine of the ramp angle
   - A skateboarder can increase his kinetic energy by bending down (reducing their radius) when they fall down a circular ramp. Here, just the conservation of momentum is abused. In skateboarder lingo, this is called 'pumping'. Source

   and any explanations about equations of motions, etc

3. **[20 marks]** A 1-dimensional particle of mass m is confined to a circle of radius R on the 2-d Euclidean plane. Find the energy eigenvalues and the normalized wave-function.

   Solution

   Equation to be solved:

   $$\frac{-\hbar^2}{2m}\nabla^2\phi = E\phi$$

   with boundary conditions:

   $$\phi(\theta) = \phi(\theta + 2\pi) \qquad \phi'(\theta) = \phi'(\theta + 2\pi)$$

   Energy eigenvalues: $E_n = \frac{n^2\hbar^2}{2mr^2}$
   Normalized eigenfunctions: $\phi_n = \frac{1}{\sqrt{2\pi}}e^{in\theta}$

END OF QUESTION PAPER