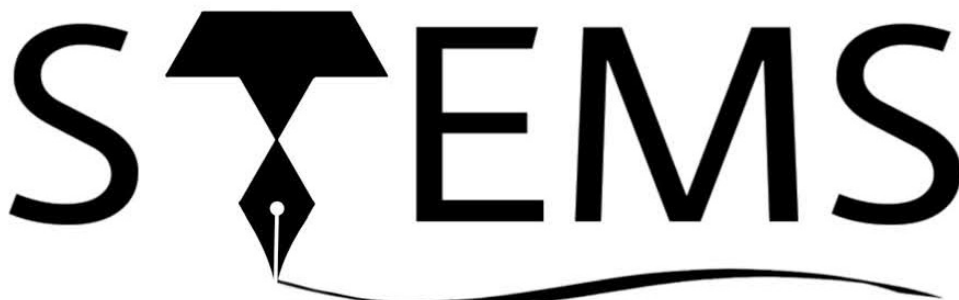




TESSELLATE PRESENTS



Scholastic Test of Excellence in Mathematical Sciences

Physics Category B

Exam Date : 16th December, 2023
Exam Timing : 9:00 AM IST - 12:00 PM IST



Rules and Regulations

Marking Scheme

1. The question paper contains **ten** questions, **seven** multiple choice type objective questions (**Part A**), and **three** subjective questions (**Part B**).
2. Each objective question has a single correct option as the answer.
3. For each objective question, **4 marks** is given for the correct answer, **0 marks** is given for no attempt, and **-1 marks** is given for an incorrect answer. Each subjective question is worth **20 marks**. The third subjective question contains a bonus question worth an additional **5 marks**.
4. A candidate's submission for the **Part B** of the exam will be checked only if they are in the top 25 candidates for **Part A**.
5. Time duration is **3 hours: 9:00 AM IST - 12:00 PM IST**.
Submit your answers on the Google form given below by **12:20 PM IST**.

Miscellaneous

1. Use the google form: <https://forms.gle/smXV8JtgPL3bgZ4BA> , to submit your answers.
2. For **Part B**, you can either LaTeX or handwrite your solutions neatly.
Submit a PDF file (either scanned or LaTeXed) **ONLY**. No other form of file submission will be accepted. Name your file "**physics.rollnumber**" (here rollnumber is the last 5 digits of the Transaction ID generated at the time of registration).
3. Make sure to keep the file size below the 10 MB limit. You can use online file compression services in case your file size exceeds 10 MB.
4. Use a good application to scan handwritten text into PDF. Kindly make sure that the answers are legible and that your furniture or flooring is not a part of the submission.
5. Solutions should be brief and should contain all the necessary details. Ambiguous or illegible answers will not gain credits. If you strike something out, strike it out properly so that it is clear to the evaluator what you want to be read. Please avoid overwriting your answers.
6. Do **NOT** post/share the questions appearing in the contest on any forums or discussion groups while the contest is live. It will result in immediate disqualification of involved candidates when caught.
7. Answers should be your own and should reflect your independent thinking process. Any form of plagiarism or failure to comply with the aforementioned regulations may lead to disqualification.
8. Responses to the google form will not be accepted after the time is up.

Contact details

- For subject related queries, clearly mention your **category (B)** in the mail or WhatsApp text.
- For **subject related** queries, contact:
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Note: Use the personal emails only if the official email is unreachable. Use WhatsApp only if absolutely necessary, otherwise email is preferred.

- For **technical** queries, contact:
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Questions

Part A

1. [4 marks] A litre of dirty water falls through a cylindrical sieve with a cross-sectional area of 15cm^2 with a filter in it. There is about 10 cm^3 of dirt in the sample, which cannot pass through the filter. The flow rate is slowed down by a factor of e^{-h} where h is the height of the dirt accumulated in the sieve. Find the time it takes for all the water to pass through the sieve if it's being poured in at $10\text{ cm}^3/\text{s}$

Options: (a) $10\sqrt{3}$ (b) $150\ln 3$ (Correct) (c) $15\sqrt{3}$ (d) $10\ln 3$

2. [4 marks] In pitch darkness, a car's headlights can illuminate a distance of 40m away. What is the maximum speed at which the car can drive so that it can come to a safe stop on seeing some obstacle in front, given that the maximum deceleration of the car is 5m/s^2 ?

Options: (a) 25m/s (b) 40m/s (c) 30m/s (d) 20m/s (Correct)

3. [4 marks] You want to tune into 91.2 MHz FM using your antique radio (they were playing nice songs on there yesterday). You know it uses the classic RLC circuit resonance to amplify only the required frequency. The inductor has an inductance of 100 nH. There is a dial that lets you change the built-in capacitance. Find the value of the capacitance.

Options: (a) 40 pF (b) 30 pF (Correct) (c) 1.2 nF (d) $5.5\ \mu\text{F}$

4. [4 marks] Batman is driving at 270 km/h when he notices Poor Percy, an officer, holding an ultrasonic gun, used for measuring the speed of a car, 1km away. He reacts within 10^{-10}s and decides to decelerate to the speed limit (90 km/h) before the ultrasonic wave hits his car. What is the constant deceleration rate required? (speed of sound is 340 m/s)

Options: (a) 22 m/s^2 (b) 10 m/s^2 (c) 100 m/s^2 (d) 11 m/s^2 (Correct)
(Edited)

5. [4 marks] Sorry but Batman is too cool to decelerate. He instead decides to accelerate to half the speed of light because he doesn't care about Poor Percy's opinion. How much energy will he need (in units of $m_b c^2$ where m_b is the mass of the vehicle)? You can use the formula:

$$E^2 = (mc^2)^2 + (pc)^2$$

Options: (a) 1.54 (b) 1.15 (c) 0.15 (Correct) (d) 0.12



The next two questions are based on the paragraph below

Alex free-falls into a pool from 10m above the surface. Let the upward force on him due to the water be $20h$ where h is the depth. His mass is 60 kg and acceleration due to gravity is 10 m/s^2 .

6. [4 marks] How much time does it take for him to stop?

Options: (a) 12.2s (b) 2.7s (c) 7s (d) 5.4s

(Question was skipped from grading because no option was correct. 5.66s is the correct answer)

7. [4 mark] To what depth does Alex reach?

Options: (a) 24.5m (b) 36.8m (c) 10m (d) 26m

(Question was skipped from grading because no option was correct. 68.7m is the correct answer)



Part B

1. [20 marks] A person throws a ball down a ledge of depth h with velocity \vec{v} and at the same time jumps up and forward with velocity \vec{w} . Find the relation between these velocities so that the person meets the ball at the same height as the initial jumping spot.

Solution

The question was supposed to be interpreted as follows:

The ball falls into the ledge and bounces back elastically, and when it reaches the same height it was thrown from, it's supposed back to meet the person falling down.

x-axis: $s = u_x t = v_x t \Rightarrow u_x = v_x$

y-axis:

$$\begin{aligned} t_w &= 2w_y/g = t_v \\ h &= v_y t + gt^2/2 \\ &= 2v_y w_y/g + 1/2g(2w_y/g)^2 \\ &= 2w_y/g(v_y + w_y) \end{aligned}$$

Any form of the last expression was accepted.

2. [20 marks] Garfield is trying to measure the radius of the earth. He sticks a pole of varying heights above the ground and notes the angle of declination you need to look at to be looking at the horizon. The table below notes his readings. Use this information to approximate the radius of Earth. Why is Garfield not doing this himself, you ask? Because it's a Saturday.

S. No.	Height [h] (in m)	Angle [θ] (in degrees)
1	10	2.5
2	20	3.5
3	30	4.5
4	40	5.0
5	50	6.0

Solution

The expression for R is:

$$R = \frac{h \cos \theta}{1 - \cos \theta}$$

The values for the five readings are: 10497, 10703, 9702, 10472, 9077, in m. The average being 10090m. Close enough answers were accepted.



3. [20(+5) marks] A point mass (mass m) is strung between two springs (spring constant k) at $(0, a)$ and $(0, -a)$ with natural length b . Find the equation of motion for small oscillations in the x component. (Ignore the y component)

Bonus: What changes when $b = a$?

Solution

$$\begin{aligned} m\ddot{x} &= -2k(\sqrt{a^2 + x^2} - b) \frac{x}{a^2 + x^2} \\ &= -2kx \left(1 - \frac{b}{\sqrt{a^2 + x^2}} \right) \\ &= -2kx \left(1 - \frac{b}{a} \frac{1}{\sqrt{1 + (x/a)^2}} \right) \\ &= -2kx \left(1 - \frac{b}{a} \left(1 - \frac{x^2}{2a^2} \right) \right) \\ &= -2k \left(1 - \frac{b}{a} \right) - \frac{bk}{a^3} x^3 \end{aligned}$$

When $b \neq a$, and x is small, the oscillation is close to linear harmonic.

When $b = a$ (or b is just very close to a), the oscillation is purely cubic harmonic.

END OF QUESTION PAPER