TESSELLATE PRESENTS

STEMS

Scholastic Test of Excellence in Mathematical Sciences

Physics Category B

Exam Date : 7th January, 2023
Exam Timing : 9 AM - 12 PM IST

Rules and Regulations

Marking Scheme

1. Time duration is **3 hours: 9 AM - 12 PM IST**. You have **20 minutes** to scan and upload your papers after that.
2. This paper contains **9 Objective** and **3 Subjective** questions. The maximum score one can obtain is **100 points**. Each objective question from $1-7$ is worth **4 points**, 8 and 9 are **4 points individually** and **12 points** if both are correct, and each subjective question is worth **20 points**. There is no negative marking.
3. **The subjective part will be graded only if you score above a certain cut-off (to be decided later) in the objective section of the paper.**
4. **The final cut-off shall be based on your total score (Objective + Subjective).**

Miscellaneous

1. Submit your answers through `https://forms.gle/TNLnPhjChcan5CMC9`.
2. Write your solutions to the subjective problems neatly, then scan and generate a PDF, which you must submit through the same form. Solutions should be brief and should contain all the necessary details. Name your file as **STEMS_Physics_Roll number.pdf**, for example, **STEMS_Physics_6900.pdf**.
3. Use a good application to scan handwritten text into PDF. Kindly make sure that the answers are legible and that your furniture or flooring is not a part of the submission.
4. Make sure your PDF has a size below 10 MB.
5. Ambiguous or illegible answers will not gain credits. If you strike something out, strike it out properly so that it is clear to the evaluator what you want to read. Please avoid overwriting your answers.
6. Answers should be your own and should reflect your independent thinking process.
7. Do **NOT** post the questions on any forums or discussion groups. It will result in immediate disqualification of involved candidates when caught.
8. Sharing/discussion aimed towards solving or distribution of problems appearing in the contest while the contest is live in any kind of online platform/forum shall be considered as a failure in complying with the regulations.
9. Any form of plagiarism or failure to comply with aforementioned regulations may lead to disqualification.
10. SI Units are used throughout unless specified otherwise.

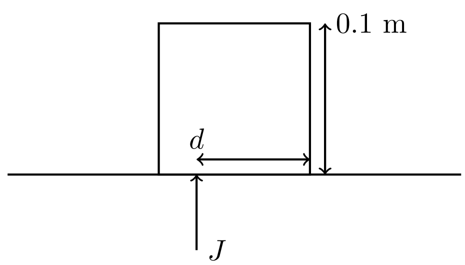
Contact details

- For subject related queries, clearly mention your **name** and **category (A)** in the mail or WhatsApp text.
- For **subject related** queries, contact **stemsphysics2023@gmail.com**. Below are two more contacts, but **please use these only if the previous one is down.**
  - Adhvik Jagannathan: **adhvik@cmi.ac.in**
  - Anand Balivada: **anandb@cmi.ac.in**
- For **technical** queries, contact **tessellate.cmi@gmail.com**. Below are two more contacts, but **please use these only if the previous one is down.**
  - Siddhant Shah: **siddhants@cmi.ac.in**
  - Rohan Goyal: **rohang@cmi.ac.in**
- Your should fill in your answers to the objective questions in the google form we sent along with this question paper. Upload the scanned PDF containing the answers to subjective questions with the forementioned file name to the same form (that contained the drive link for the question paper) along with your **name, subject, category and your registered email ID** on it. Submissions by emails will be accepted only till **12:20 PM IST.**

Questions

Objective

1. The hit game geometry dash involves a square of mass $4kg$, length $0.1m$ and moment of inertia $0.005\ kgm^2$ about its center of mass. The square jumps over two dimensional obstacles as it moves with a constant speed. If the highest it can jump is twice its height, and the number of rotations it undergoes in one jump is half, determine the impulse J it applies on the ground in order to jump, and the distance d between the front of the cube and its (average) point of application, in S.I. units. **Input your final answer as $J + 10d$.** You may approximate using the greatest integer function. Gravity is defined by $g = 10\ ms^{-2}$ in this game.



2. To make preparations for Blue Lock, Ego Jinpachi needs to test out the iconic pentagonal football stadium. He does this by launching N identical footballs ($N >> 1$) of $0.4kg$, each moving randomly at a speed of $5ms^{-1}$. The stadium is a regular pentagon with side length $100m$ and height $10m$. Assuming all collisions of the balls with each other and the walls are elastic and neglecting friction, **determine the average force exerted by the balls on any of the vertical sides of the stadium.**

   (a) 20 N.

   (b) $\dfrac{20}{\sqrt{5(5+2\sqrt{5})}}$ N.

   (c) 10 N.

   (d) $\dfrac{10}{3\sqrt{3}}$ N.

   Hint: The number of "particles" is "large" and the motions are random.

3. Consider an ideal gas of N particles at $1000K$ confined to a rigid container of surface area A, with coefficient of emissivity e, with the surroundings maintained at $200K$. The ideal gas loses its internal energy and experiences a drop in temperature due to radiation. If $\alpha = \frac{\sigma e A}{c_V N k_B}$, **which is the best estimate for the amount of time it takes for the gas to attain 900 K?**

   (a) $6.125 \cdot 10^{-9} \alpha^{-1}\ s$.

   (b) $5.7 \cdot 10^{-10} \alpha\ s$.

   (c) $1.242 \cdot 10^{-10} \alpha^{-1}\ s$.

   (d) $9.966 \cdot 10^{-11} \alpha^{-1}\ s$.

4. A (cylindrical) nail of radius r is hammered into a wooden plank by a depth H in the summer. During the winter, the wood contracts, and thus applies stress on the nail, which varies with the depth as:

$$\sigma(h) = \Lambda h(H - h)$$

   If the coefficient of friction between the nail and the wooden plank is μ, **what is the minimum constant force required to pull the nail out during winter?**

   (a) $\dfrac{\pi \mu \Lambda r H^3}{4}$.

   (b) $\dfrac{\pi \mu \Lambda r H^3}{3}$.

   (c) $\dfrac{2\pi \mu \Lambda r H^3}{3}$.

   (d) $\dfrac{3\pi \mu \Lambda r H^3}{4}$.

5. A proton and neutron (you can take both to be of mass m and proton with charge e), each travelling at speed v in straight lines, collide at a point P, their trajectories making an angle θ with each other. Immediately after their collision, a magnetic field with strength B, perpendicular to the plane of the particles' motion is switched on throughout space. Before collision, the particles' center of mass travelled along a line at constant speed. **What is the time T after the collision at which the trajectory of the center of mass of the system intersects this line**?

   (a) $T = \frac{m}{eB \sin \theta/2}$.

   (b) This happens if and only if $v = \frac{eB}{m}$, and $T = \frac{\pi m}{2eB}$.

   (c) This never happens after the collision.

   (d) $T = \frac{m}{eB}$.

6. White light is used to illuminate the two slits in Young's double slit experiment. The separation between the slits is d and the distance between the screen and the slit is D ($>> d$). At a point on the screen directly in front of one of the slits, **what are the missing wavelengths?** Here $m \in \mathbb{N}$.

   (a) $\lambda = \dfrac{d^2}{(2m+1)D}$.

   (b) $\lambda = \dfrac{(2m+1)d^2}{D}$.

   (c) $\lambda = \dfrac{d^2}{(m+1)D}$.

   (d) $\lambda = \dfrac{(m+1)d^2}{D}$.

7. A point source of light S is placed at the bottom of a vessel containing a liquid of refractive index $\frac{2}{\sqrt{3}}$. There is an opaque disc of radius r floating on the surface, with its center directly above S. The liquid is gradually drained out from the vessel. **The maximum height of the liquid at which the source cannot be seen by an observer above the surface is**

   (a) r

   (b) $\sqrt{3}r$

   (c) $\frac{r}{\sqrt{3}}$

   (d) $2r$

Questions 8 and 9 are based on the paragraph below.

The existence of electric monopoles (i.e. charges) is well known. However, magnetic monopoles are purely hypothetical in physics. If they did exist, there exist some symmetries between the equations which apply to electric and magnetic charged particles. You are allowed to look these up, as you are for any other information.

8. A thin circular loop contains a uniform current of magnetic charges. If an electric dipole was brought towards the loop at a constant speed along the axis of the loop, with its positive end facing the loop, **which of the following are true?**

   (a) The magnetic current would increase if the initial current was clockwise from the direction of the dipole.

   (b) The magnetic current would decrease if the initial current was clockwise from the direction of the dipole.

   (c) A magnetic current would be induced in the clockwise direction if the initial current was zero.

   (d) The magnetic current would not change regardless.

9. If a magnetic charge initially moved at a velocity v in the $\hat{\mathbf{x}}$ direction, and at time $t = 0$ a uniform electric field $\mathbf{E} = E_0\hat{\mathbf{y}}$ is switched on, **what is the motion undergone by the charge after that?**

   (a) The particle goes in a uniform circular path with its angular momentum in the $\hat{\mathbf{y}}$ direction.

   (b) The particle goes in a uniform circular path with its angular momentum in the $-\hat{\mathbf{y}}$ direction.

   (c) The particle goes in a uniform circular path with its angular momentum in the $\hat{\mathbf{z}}$ direction.

   (d) The particle continues to move in a straight line, with a constant acceleration.

Subjective

1. Assume no relativistic effects. Two particles of masses m_1, m_2, initially at a distance d away from each other, are set free. **Find the time elapsed as a function $t(x_1)$ of the distance x_1 of one of the bobs from the center of mass of the system, for their journey before they "meet".** Using this, **graph x_1 as a function of time, and explain its qualitative features in detail.** You may use suitable graphing software, but **mention** it if you did.

2. A water heater has a coil of resistance R, inductance L and efficiency 80%, and is installed in a household providing 240 V DC. The heater is imported, and is meant to work under a DC voltage of 120 V, which is why a step-down adapter is required to bring it up from 120 V to 240 V. The adapter can change its voltage from 120 V to 240 V and back at the flick of a switch, without interrupting the circuit.

   Suppose there are two heaters,with steady currents flowing through them. One of them has the adapter on, while the other one has it off. At $t = 0$, the adapter is switched on. If $\Delta T_1(t)$ and $\Delta T_2(t)$ are the increases in temperatures of 1kg of water after being subject to heating by each heater for time t, **calculate** $\lim_{t\to\infty}[\Delta T_1(t) - \Delta T_2(t)]$. The specific heat capacity of water is 4200 Jkg^{-1}.

3. You are tasked with manufacturing a chip designed by the evil LeeChant Corporation using electron beam lithography. This involves focusing an electron beam on the semiconductor wafer to draw out LeeChant's diabolical circuits.

   The first batch that you produce is extremely faulty, leading to LeeChant's phones blowing up and them having to recall 1 million of them! Naturally, they've taken you hostage and are forcing you to fix their chips.

   Your team figures out that the electron beam is distorted by the thermal motion of the electrons and model its current density with the Gaussian

$$J = J_p e^{-\frac{r^2}{\sigma^2}} \qquad (r > 0)$$

   $d_G = 2\sigma$ is the effective diameter of the beam. Your first task is -

   (a) **Find a relationship between the total current I in the beam and d_G.**

   There is good news on the horizon! You can us a lens to focus the beam towards a point, with it making an angle of α with the axis of the lens given by -

$$J_p = J_c \left(1 + \frac{eV_0}{kT_c}\right) \sin^2 \alpha$$

   with the current density of the source J_c, voltage of the electrons V_0 and the temperature corresponding to the electron energy T_c ($T_c < 10^3 K$ and $V_0 \geq 5 \cdot 10^4 V$).

(b) For small α, **determine an upper bound for d_G in terms of α, I, V_0, J_c and T_c.**

Your team, armed with this knowledge, is able to tune the parameters of the electron beam to LeeChant's liking, and none of their phones after that seem to be blowing up!

$k \approx 1.38 \cdot 10^{-23} JK^{-1}$ denotes Boltzmann's constant, and $e \approx 1.6 \cdot 10^{-19} C$ denotes the electronic charge.

All the Best!!