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**Q1** Let $f(x)=x^{2021}+15x^{2020}+8x+9$ have roots $a_i$ where $i=1,2,\cdots,2021$. Let $p(x)$ be a polynomial of the sam degree such that $p\left(a_i+\frac{1}{a_i}+1\right)=0$ for every $1\leq i\leq 2021$. If $\frac{3p(0)}{4p(1)}=\frac{m}{n}$ where $m,n\in\mathbb{Z}$, $n>0$ and $\gcd(m,n)=1$. Then find $m+n$.

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**Q2** Suppose $f:\mathbb{R}^+\mapsto\mathbb{R}^+$ is a function such that $\frac{f(x)}{x}$ is increasing on $\mathbb{R}^+$. For $a,b,c>0$, prove that

$$2\left(\frac{f(a)+f(b)}{a+b}+\frac{f(b)+f(c)}{b+c}+\frac{f(c)+f(a)}{c+a}\right)\geq 3\left(\frac{f(a)+f(b)+f(c)}{a+b+c}\right)+\frac{f(a)}{a}+\frac{f(b)}{b}+\frac{f(c)}{c}$$

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**Q3** An acute scalene triangle $\triangle ABC$ with altitudes $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ is inscribed in circle $\Gamma$. Medians from $B$ and $C$ meet $\Gamma$ again at $K$ and $L$ respectively. Prove that the circumcircles of $\triangle BFK, \triangle CEL$ and $\triangle DEF$ concur.

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**Q4** Let $n>1$ be any integer. Define $f,g$ as functions from $\{0,1,2,\cdots,n-1\}$ to $\{0,1,2,\cdots,n-1\}$ defined as

$$f(i)=2i \pmod n$$
$$g(i)=2i+1 \pmod n$$

Show that for any integers $\ell,m\in\{0,1,2,\cdots,n-1\}$ , there are infinitely many compositions of $f,g$ that map $\ell$ to $m$

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**Q5** Let $ABC$ be a triangle with $I$ as incenter.The incircle touches $BC$ at $D$.Let $D'$ be the antipode of $D$ on the incircle.Make a tangent at $D'$ to incircle.Let it meet $(ABC)$ at $X,Y$ respectively.Let the other tangent from $X$ meet the other tangent from $Y$ at $Z$.Prove that $(ZBD)$ meets $IB$ at the midpoint of $IB$

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