STEMS

Scholastic Test of Excellence in Mathematical Sciences

Computer Science Category A

Exam Date : 25th January 2020
Exam Timing : 12noon - 3pm

Rules and Regulations

Marking Scheme

1. The question paper is divided in two parts -
   Objective (8 questions) + Subjective (5 questions).

2. Each objective question is worth **2 points**.

3. Each subjective problem is worth **10 points**.

4. There is no negative marking in any section.

5. **The subjective part will be graded only if you score above a certain cut-off (to be decided later) in the objective section of the paper**.

6. **For the final score, your total score (subjective + objective) will be taken into consideration.**

Solution guidelines

1. You are **NOT** required to show your work for the objective part of the paper.
   Only tick your option choices in a tabular format drawn on a blank sheet.
   A sample is shown below.

Q. No.	(a)	(b)	(c)	(d)
1		✓		
2			✓	
3		✓		
4	✓			
5				✓

Fig. - Sample objective answer submission

2. Provide a complete solution/proof for subjective questions. The solutions must be correct, original, detailed and clearly understandable for full credit.
   Partial credit might be awarded to incomplete proofs, based on the progress made towards solving the problem.

3. Draw clear, well-labeled diagrams wherever necessary.

Miscellaneous

1. Answers should be your own and should reflect your independent thinking process.

2. Do **NOT** post the questions on any forums or discussion groups. It will result in immediate disqualification of involved candidates when caught.

3. Answers should be written clearly, in a legible way. Formal proofs are required wherever asked for. Unclear reasoning might not be awarded points, draw clear diagrams wherever necessary.

4. Sharing/discussion aimed towards solving or distribution of problems appearing in the contest while the contest is live in any kind of online platform/forum shall be considered as a failure in complying with the regulations.

5. Any form of plagiarism or failure to comply with aforementioned regulations may lead to disqualification.

Contact details - ONLY for subject related queries

- Please do not call these people for technical problems or submission inquiries.
  Only if you find an ambiguity in a question and need clarification, use these contacts.

- As our phone numbers will be busy, **we prefer WhatsApp & email queries** instead. Only call us if absolutely necessary.

- Try to solve all your submission related doubts from information in the next page ONLY. We have included all details in the next section.

Debjit Paria - 8637077315
Query email - stems.2020.enq.cs@gmail.com

- **Do NOT call any number to ask if your submission has reached us.**
  If you send your submission to the right email address with mentioned details, we will receive it. We will contact all participants who fail to submit, so please be patient.

How to submit your answers

1. Write the following details **as per your registration** on the first page of your submission file/photographs -

Name	:	Your full name
School/College name	:	
Class/Year of Study	:	Class 8, Class 11, Undergrad 1st year etc.
Registered Email address	:	
Mode of S.T.E.M.S. Registration	:	Online / Through School / Other (mention details)

2. Write all your answers on sheets of paper, following all the solution guidelines. Write the page number in the top right corner of every sheet.

3. Scan your answers or take clear photographs of your response sheets. Compile them into a single PDF file or send all pictures in the right sequence.

   If you have a limited file size issue when sending your submission, make a new Google Drive folder with title '**Your Name - Computer Science A submission**'.

4. Send the submission file or Google Drive link from your registered email -

Submission email address	:	**stems.2020.cs.a@gmail.com**
Subject of email	:	Computer Science Category A Submission - STEMS
Submission deadline	:	25th January 2020 - 15:00

Good luck, happy problem-solving!

Objective Questions

For **Problems 1-8**, each problem has **four** options, namely **(a), (b), (c), (d)**, of which *only one* is correct, **2 point** will be awarded for correctly answering a problem, **NO** negative marks shall be awarded for wrong answers/unattempted problems.

Problem 1. *How many sorts will the Bubble-sort algorithm do to sort the following array:*
$\{6, 3, 1, 4, 2, 5\}$
a. 7
b. 8
c. 9
d. 10

Problem 2. *Consider the following function:*
$f(1) = 0$
$f(n) = 1 + f(n/p)$ *where p is the largest prime divisor of n.*
What is $f(12600)$?
a. 5
b. 6
c. 7
d. 8

Problem 3. *What is the number of paths from $(0,0)$ to $(10,10)$ which can only go from (i, j) to $(i+1, j)$ or $(i, j+1)$ which do not ever cross the diagonal (that is the path doesn't consist of cells (x, y) with $y > x$)?*
a. 16384
b. 58786
c. 16796
d. 4862

Problem 4. *Which of the following expressions is equivalent to the given summation:*

$$\sum_{i=0}^{\lfloor N/2 \rfloor} \binom{N-i}{i}$$

a. $\binom{2N}{N}$

b. $fibonacci(N + 1)$ *(refer to $https://en.wikipedia.org/wiki/Fibonacci\_number$)*

c. 2^N

d. $catalan(n)$ *(refer to $https://en.wikipedia.org/wiki/Catalan\_number$)*

Problem 5. *What is the number of ways of collecting Rs. 16 using coins of Rs. $1, 2, 5, 10$?*

a. 18

b. 20

c. 24

d. 25

Problem 6. *Count the number of ways to fill an array of size n with integers such that minimum of all elements in array is 1 and maximum of elements in array is k.*

a. $(k^n - 2)(k - 1)^n + (k - 2)^n$

b. $(k - 1) \times k^{n-1}$

c. $k^n - 2(k - 1)^n + (k - 2)^n$

d. $(k - 2) \times k^{n-1}$

Problem 7. *What will be the result of this code:*

```
mod = 10^9 + 7;
slowpower(a , b){
    result = 1;
    for(i = 0 ; i < b ; ++i) {
        result = (result * a) % mod;
    }
     return result;
}

print(slowpower(20202019, 420696961231))
```

a. 689950549

b. 683210549

c. 689713549

d. 669150549

Problem 8. *Which of the following can NOT be derived from the given statements:*
I. All tigers are animals.

II. All animals are alive.
III. All who are alive eat food.
IV. No tiger is female.
a. All tigers eat food and they are alive.
b. A male is not a tiger.
c. Those who don't eat food are not tiger.
d. All males eat food.

Subjective Problems

Problem 1. *Consider a partition of the 52 cards of a deck into 13 groups of 4 cards each. Prove that there is a way to choose 1 card from each group such that all 13 distinct cards show up.*

Problem 2. $A = \{1, 20, 38, 69, 73, 77, 83, 96, 98, 199\}$

Two numbers a and b said to be comparable iff a&b = b or a&b = a where & is the bitwise and operation. Find a partition of A with the minimal number of classes such that no two elements in a class are comparable.

Problem 3. *Show that if 15 distinct numbers are chosen from the set $\{1, 2, ..., 45\}$, there will exist a pair of numbers differeing by either 1, or 3, or 4.*

Problem 4. *Consider a pentagon $A_1 A_2 A_3 A_4 A_5$. Duppy the puppy, sits at the vertex A_3 of the pentagon. His favorite dog biscuits, which he loves to have for dinner, has been kept at the vertex A_1. However, poor Duppy is blind and does not have a keen sense of smell. So, every minute, he moves from the current vertex he is at to one of the adjacent vertices with equal probability. Thus, after one minute, Duppy is at vertex A_2 or vertex A_4, each with probability $\frac{1}{2}$. Also, it has no memory of the vertex it is at, at any time. So, after the second minute, it might have returned to the vertex A_3.*

Compute the mean and variance of the number of minutes till Duppy gets his dinner.

Problem 5. *Consider a constant γ in the range $0 < \gamma < \frac{1}{2}$. Show that the following asymptotic formula holds :*

$$\sum_{i \leq \gamma n} \binom{n}{i} = 2^{n\mathcal{H}(\gamma) - \frac{1}{2}\log n + O(1)}$$

where $\mathcal{H}(x) = x \log \frac{1}{x} + (1-x) \log \left(\frac{1}{1-x}\right).$